TWO CLASSES OF SOLUTIONS OF THE GASDYNAMICAL EQUATIONS

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1. A class of solutions of the unsteady three-dimensional gasdynamical equations has been found [1] such that the components of the velocity vector depend linearly on all the space coordinates x_1, x_2, x_3. These solutions are described by a system of ordinary differential equations with an independent time variable t and have been used, in particular, to study the dynamics of a gravitating gas ellipsoid [2]. Certain solutions of the Navier–Stokes equations for steady three-dimensional flows of an incompressible viscous fluid with a linear dependence of the components u_i of the velocity vector **u** on two coordinates x_1, x_2, and with a special pressure function p are described in [3].

Below, we investigate the solutions of the dynamical equations for an ideal gas, where u_i, the entropy function W, and the function $Q = \rho^{\gamma-1}$ (in which ρ is the density and γ is the adiabatic exponent in the equation of state $p = W\rho^{\gamma}$) depend linearly on some of the space coordinates. Unlike the situation in [1], the investigation of motions of this type is reducible in general to the analysis of the compatibility of overdetermined systems of partial differential equations having a complex structure.

We propose to set forth certain sufficient conditions whereby the corresponding overdetermined systems are reducible to determinate systems, for which the number of equations is the same as the number of unknown functions and which have sufficient arbitrariness in the solutions. In this way we establish the nonemptiness of the given classes of solutions and then, by the construction of concrete examples, demonstrate the substantiality of these classes. The complete compatibility analysis of the overdetermined systems and the classification of the solutions remain open questions.

In the absence of external forces the system of gasdynamical equations is written in the form

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\nabla)\mathbf{u} + Q \operatorname{grad} W + \frac{\gamma}{\gamma-1} W \operatorname{grad} Q = 0; \tag{1.1}$$

$$\frac{\partial Q}{\partial t} + (\mathbf{u} \operatorname{grad} Q) + (\gamma-1) Q \operatorname{div} \mathbf{u} = 0; \tag{1.2}$$

$$\frac{\partial W}{\partial t} + (\mathbf{u} \operatorname{grad} W) = 0. \tag{1.3}$$

A. Linearity in One Coordinate. We seek solutions of the system (1.1)-(1.3) in the form

$$u_i = f_i(x_1, x_2, t)x_3 + g_i(x_1, x_2, t); \tag{1.4}$$

$$Q = f(x_1, x_2, t)x_3 + g(x_1, x_2, t); \tag{1.5}$$

$$W = F(x_1, x_2, t)x_3 + G(x_1, x_2, t). \tag{1.6}$$

Substituting expressions (1.4)-(1.6) into the system (1.1)-(1.3), we reduce each equation of the system to the form

$$A_i x_3^2 + B_i x_3 + C_i = 0, \quad i = 1, 2, \ldots, 5,$$

where the functions A_i, B_i, C_i are expressed in terms of the coefficients of the representations (1.4)-(1.6) and their derivatives and depend only on x_1, x_2, t. Setting

$$A_i = B_i = C_i = 0 \tag{1.7}$$

by the arbitrariness of x_3, we obtain a system of 15 partial differential equations in 10 unknown functions f_i, g_i, f, g, F, G.

We introduce the two-dimensional vectors $\mathbf{v} = (f_1, f_2)$, $\mathbf{w} = (g_1, g_2)$, $\mathbf{R} = (g, f)$, $\mathbf{S} = (F, G)$, $\mathbf{P}_f = (F, f)$, $\mathbf{P}_g = (G, g)$, $\mathbf{T}_j = (f, (\gamma/\gamma - 1)F)$, $\mathbf{T}_g = (g, (\gamma/\gamma - 1)G)$. Then the system (1.7) can be represented in the form

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$$(\mathbf{v}\cdot\nabla)\mathbf{v}+\left(\mathbf{T}_f\cdot\frac{\partial\mathbf{P}_f}{\partial x_i}\right)=0,\quad i=1,2, \tag{1.8}$$

$$(\mathbf{v}\cdot\operatorname{grad} f_3)=0,\ (\mathbf{v}\cdot\operatorname{grad} f)+(\gamma-1)f\operatorname{div}\mathbf{v}=0,$$
$$(\mathbf{v}\cdot\operatorname{grad} F)=0;$$

$$\frac{\partial\mathbf{v}}{\partial t}+(\mathbf{v}\nabla)\mathbf{w}+(\mathbf{w}\nabla)\mathbf{v}+f_3\mathbf{v}+\left(\mathbf{R}\cdot\frac{\partial\mathbf{S}}{\partial x_i}\right)+\frac{\gamma}{\gamma-1}\left(\mathbf{S}\cdot\frac{\partial\mathbf{R}}{\partial x_i}\right)=0,\quad i=1,2, \tag{1.9}$$

$$\frac{\partial f_3}{\partial t}+(\mathbf{v}\operatorname{grad} g_3)+(\mathbf{w}\operatorname{grad} f_3)+f_3^2+(\mathbf{P}_f\cdot\mathbf{T}_f)=0,$$

$$\frac{\partial f}{\partial t}+(\mathbf{v}\operatorname{grad} g)+(\mathbf{w}\operatorname{grad} f)+\gamma f f_3+(\gamma-1)f\operatorname{div}\mathbf{w}+(\gamma-1)g\operatorname{div}\mathbf{v}=0$$

$$\frac{\partial F}{\partial t}+\mathbf{v}(\operatorname{grad} G)+(\mathbf{w}\operatorname{grad} F)+Ff_3=0;$$

$$\begin{gathered}\partial\mathbf{w}/\partial t+(\mathbf{w}\nabla)\mathbf{w}+g_3\mathbf{v}+(\mathbf{T}_g\cdot\partial\mathbf{P}_g/\partial x_i)=0,\ i=1,2,\\ \partial g_3/\partial t+(\mathbf{w}\operatorname{grad} g_3)+f_3g_3+(\mathbf{T}_g\cdot\mathbf{S})=0,\\ \partial g/\partial t+(\mathbf{w}\operatorname{grad} g)+\gamma f g_3+(\gamma-1)g\operatorname{div}\mathbf{w}=0,\\ \partial G/\partial t+(\mathbf{w}\operatorname{grad} G)+Fg_3=0.\end{gathered} \tag{1.10}$$

Equations (1.8)-(1.10) are the respective systems $A_i = 0$, $B_i = 0$, and $C_i = 0$.

In the system (1.8)-(1.10) we set

$$f_1\equiv f_2\equiv f\equiv F\equiv 0.$$

Then $\mathbf{v} = \mathbf{P}_f = \mathbf{T}_f = (\mathbf{R}\cdot\partial\mathbf{S}/\partial x_i) = (\mathbf{S}\cdot\partial\mathbf{R}/\partial x_i) = 0$, and all the equations (1.8)-(1.9) except the one containing $\partial f_3/\partial t$ will be satisfied. There now remain six equations in six unknown functions g_1, g_2, g_3, f_3, g, G. Finally, we write this determinate system in the form

$$\begin{gathered}\frac{\partial f_3}{\partial t}+(\mathbf{w}\operatorname{grad} f_3)+f_3^2=0,\\ \frac{\partial\mathbf{w}}{\partial t}+(\mathbf{w}\nabla)\mathbf{w}+g\frac{\partial G}{\partial x_i}+\frac{\gamma}{\gamma-1}G\frac{\partial g}{\partial x_i}=0,\quad i=1,2,\\ \partial g_3/\partial t+(\mathbf{w}\operatorname{grad} g_3)+f_3g_3=0,\\ \partial g/\partial t+(\mathbf{w}\operatorname{grad} g)+(\gamma-1)g(f_3+\operatorname{div}\mathbf{w})=0,\\ \partial G/\partial t+(\mathbf{w}\operatorname{grad} G)=0.\end{gathered} \tag{1.11}$$

We note that the case $\mathbf{v} = 0$, $(\mathbf{T}_f\cdot\mathbf{P}_f/\partial x_i) = 0$, when $Fg^{\gamma/\gamma-1} = L(t)$ [$L(t)$ is an arbitrary function], yields an overdetermined system of seven equations in six functions g_i, f_3, F, G.

From the system of Eq. (1.11) we at once obtain equations describing certain special cases.

1. Putting $g_2 = 0$ and assuming that all the unknown functions depend only on x_1 and t, we obtain a system of five equations for plane-parallel (in the x_1, x_2 plane) flows with a linear dependence of the principal functions on x_3.

2. Putting $\partial/\partial t = 0$ in (1.11), we obtain a system of equations for steady three-dimensional flows.

3. For the elementary case of plane-parallel steady flow we write out the system of ordinary differential equations for the functions $g_1(x_1)$, $g_3(x_1)$, $f_3(x_1)$, $g(x_1)$, $G(x_1)$:

$$\begin{gathered}g_1f_3'+f_3^2=0,\quad g_1g_1'+gG'+\frac{\gamma}{\gamma-1}Gg'=0,\\ g_1g_3'+f_3g_3=0,\quad g_1g'+(\gamma-1)g(g_1'+f_3)=0,\quad g_1G'=0.\end{gathered} \tag{1.12}$$

It follows from the first and third equations (1.12) that $g_3 = Cf_3$, $C = \text{const}$, and without loss of generality we can put $C = 0$, because C determines the shift of the origin along the x_3 axis. In the given situation, therefore, we have a class of isentropic planar flows ($G = G_0 = \text{const}$), which is specified by the following relations after integration of the second equation (1.12):

$$g_1f_3'+f_3^2=0,\ \frac{1}{2}g_1^2+\frac{\gamma}{\gamma-1}G_0g=K_0>0\ (K_0=\text{const}),\quad g_1g'+(\gamma-1)g(g_1'+f_3)=0. \tag{1.13}$$

Expressing the functions g and g_1 in terms of f_3, for f_3 we obtain a second-order equation that does not contain x_1. Integrating it, we have

$$f_3'^{\frac{\gamma+1}{\gamma-1}}=K_2\left(K_1f_3'^2-\frac{\gamma-1}{2}f_3^4\right)^{\frac{1}{\gamma-1}}f_3,\quad K_1=(\gamma-1)K_0,\quad K_2=\text{const}. \tag{1.14}$$

Again the variable x_1 does not enter into (1.14), so that the system (1.12) is integ...ble in quadratures.

B. Linearity in Two Coordinates. We seek solutions of the system (1.1)-(1.3) in the form

$$
\begin{aligned}
u_i &= l_i(x_1, t)x_2 + f_i(x_1, t)x_3 + g_i(x_1, t), \ i = 1, 2, 3, \\
Q &= l(x_1, t)x_2 + f(x_1, t)x_3 + g(x_1, t), \\
W &= L(x_1, t)x_2 + F(x_1, t)x_3 + G(x_1, t).
\end{aligned} \tag{1.15}
$$

Substituting (1.15) into (1.1)-(1.3), we obtain five relations of the type

$$
Ax_2^2 + Bx_2x_3 + Cx_3^2 + Dx_2 + Ex_3 + F = 0, \tag{1.16}
$$

the coefficients of which depend only on x_1, t. Setting them equal to zero, we obtain an overdetermined system of 30 partial differential equations in 15 unknown functions from (1.15).

We set

$$
l_1 \equiv f_1 \equiv l \equiv f \equiv 0.
$$

Then all the coefficients of squared terms in relations (1.16) vanish, leaving a system of 15 equations in 11 unknown functions.

In addition, setting

$$
L \equiv F \equiv 0,
$$

we finally obtain a determinate system of nine equations in nine functions l_2, l_3, f_2, f_3, g_1, g_2, g_3, g, G. It can be written in the form

$$
\begin{gathered}
\frac{\partial l_i}{\partial t} + g_1 \frac{\partial l_i}{\partial x_1} + l_2 l_i + l_3 f_i = 0, \\
\frac{\partial f_i}{\partial t} + g_1 \frac{\partial f_i}{\partial x_1} + f_2 l_i + f_3 f_i = 0, \quad \frac{\partial g_i}{\partial t} + g_1 \frac{\partial g_i}{\partial x_1} + g_2 l_i + g_3 f_i = 0, \\
\frac{\partial g_1}{\partial t} + g_1 \frac{\partial g_1}{\partial x_1} + g \frac{\partial G}{\partial x_1} + \frac{\gamma}{\gamma - 1} G \frac{\partial g}{\partial x_1} = 0, \\
\frac{\partial g}{\partial t} + g_1 \frac{\partial g}{\partial x_1} + (\gamma - 1) g \left(\frac{\partial g_1}{\partial x_1} + l_2 + f_3 \right) = 0, \quad \frac{\partial G}{\partial t} + g_1 \frac{\partial G}{\partial x_1} = 0.
\end{gathered} \tag{1.17}
$$

It follows from (1.17) that steady three-dimensional flows of the type (1.15) can only be isentropic with $G = G_0 =$ const and are described by a system of eight ordinary differential equations.

All of the investigated flows A and B are rotational ($\partial u_1/\partial x_3 \neq \partial u_3/\partial x_1$), and in the steady-flow cases the constant in the Bernoulli integral depends on the particular streamline and changes in transition to another streamline.

2. We now consider the feasibility of constructing shock solutions for the flow classes A and B. Let us suppose that a shock wave S moves through a gas whose state is described by the system (1.11) or (1.17) and that the postshock motion of the gas belongs to class A or B, respectively. It is clear that if the motion of the shock front is described by the general equation $\Phi(x_1, x_2, t) = x_3$, then in case A the five scalar Hugoniot conditions, which must be satisfied along the surface S together with Eqs. (1.11) on both sides S, yield and overdetermined system of 17 equations in 13 unknown functions depending on x_1, x_2, t (the function Φ can be regarded as unknown). We therefore assume that the motion of the surface of S is described by the equation $\Psi(x_1, x_2, t) = 0$, i.e., at every instant the shock wave represents a cylindrical surface in x_1, x_2, x_3 space.

The Hugoniot conditions for our case have the form [4]

$$
[\rho (u_n - D)] = 0, \quad \left[\frac{1}{2} (u_n - D)^2 + \frac{\gamma p}{\rho (\gamma - 1)} \right] = 0, \tag{2.1}
$$

$$
[p + \rho (u_n - D)^2] = 0;
$$

$$
[(\mathbf{u} \cdot \boldsymbol{\tau}_1)] = 0, \ [(\mathbf{u} \cdot \boldsymbol{\tau}_2)] = 0, \tag{2.2}
$$

where u_n is the velocity component normal to the surface S, D is the normal velocity of propagation of the shock wave, and τ_1, τ_2 are orthogonal unit vectors situated in the plane tangent to S in x_1, x_2, x_3 space.

Following are the expressions for u_n, D, τ_1, and τ_2:

$$u_n = (g_1\Psi_1 + g_2\Psi_2)(\Psi_1^2 + \Psi_2^2)^{-1/2}, \quad D = -\Psi_t(\Psi_1^2 + \Psi_2^2)^{-1/2},$$
$$\tau_1 = (0, 0, 1), \quad \tau_2 = (\Psi_2, -\Psi_1, 0)(\Psi_1^2 + \Psi_2^2)^{-1/2}, \quad \Psi_k = \frac{\partial\Psi}{\partial x_k}.$$

It follows from the first equation (2.2) that $[u_3] = 0$, i.e., by the arbitrariness of x_3

$$[f_3] = [g_3] = 0. \tag{2.3}$$

The remaining Hugoniot conditions yield the relations

$$\left[g^{\frac{1}{\gamma-1}}(g_1\Psi_1 + g_2\Psi_2 + \Psi_t)\right] = 0,$$
$$\left[\frac{1}{2}\frac{(g_1\Psi_1 + g_2\Psi_2 + \Psi_t)^2}{\Psi_1^2 + \Psi_2^2} + \frac{\gamma}{\gamma-1}Gg\right] = 0, \tag{2.4}$$
$$\left[Gg^{\frac{\gamma}{\gamma-1}} + g^{\frac{1}{\gamma-1}}\frac{(g_1\Psi_1 + g_2\Psi_2 + \Psi_t)^2}{\Psi_1^2 + \Psi_2^2}\right] = 0, \quad [g_1\Psi_2 - g_2\Psi_1] = 0.$$

We thus in fact have the Hugoniot conditions for ordinary two-dimensional flows. According to the background values of g_1, g_2, g, G, g_3, f_3 specified on one side of the shock and the specified function Ψ, conditions (2.4) together with the continuity conditions on the component u_3 (2.3) determine the initial conditions for all six unknown functions of the system (1.11) along the noncharacteristic surface $\Psi = 0$. Consequently, the construction of flows in class A with shock waves is indeed possible. A concrete example of such a flow is discussed below.

Let us now suppose that for the class of solutions B the shock front is plane and that its motion is specified by the equation $\Psi(x_1, t) = 0$ (an analysis of a more general specification of the shock yields overdetermined systems of equations at a discontinuity). In the given situation we have $u_n = g_1$, $D = -\Psi_t\Psi_1^{-1}$, $\tau_1 = (0, 0, 1)$, $\tau_2 = (0, 1, 0)$. From conditions (2.2), by the arbitrariness of x_2 and x_3, we obtain (the velocity components u_2 and u_3 are continuous)

$$[l_2] = [f_2] = [g_2] = [l_3] = [f_3] = [g_3] = 0. \tag{2.5}$$

Conditions (2.1), on the other hand, have the form

$$\left[g^{\frac{1}{\gamma-1}}(g_1 + \Psi_t)\right] = 0,$$
$$\left[\frac{1}{2}(g_1 + \Psi_t)^2 + \frac{\gamma}{\gamma-1}gG\right] = 0, \quad \left[Gg^{\frac{\gamma}{\gamma-1}} + g^{\frac{1}{\gamma-1}}(g_1 + \Psi_t)^2\right] = 0. \tag{2.6}$$

As in case A, having specified the background values of the functions of the class of solutions B and the function Ψ, we can then use (2.5) and (2.6) to find the initial data on S for the system (1.17). Thus, the construction of discontinuous solutions of class B with plane shock waves is possible.

3. Here we first investigate in greater detail solutions of the form

$$u_1 = g_1(x_1),\ u_3 = f_3(x_1)x_3,\ u_2 \equiv 0,\ Q = g(x_1),\ W = G_0,$$

described by the system (1.13).

Without sacrificing generality, we choose a system of units such that $K_0 = (\gamma - 1)^{-1}$ in (1.13). Then we can represent the solution of Eq. (1.14) in the following parametric form ($K_2 > 0$, λ is the parameter):

$$f_3 = K_2\left(\lambda^2 - \frac{\gamma-1}{2}\right)^{\frac{1}{\gamma-1}}\lambda^{-\frac{\gamma+1}{\gamma-1}},$$
$$x_1 = -\frac{1}{K_2}\int \lambda^{\frac{3-\gamma}{\gamma-1}}\left(\lambda^2 - \frac{\gamma-1}{2}\right)^{\frac{\gamma}{\gamma-1}}\left(\lambda^2 - \frac{\gamma+1}{2}\right)d\lambda. \tag{3.1}$$

Integrating the streamline equation

$$dx_1/g_1 = dx_3/x_3f_3,$$

in which g_1 is expressed in terms of f_3 by means of (1.13), we have along a given streamline

$$f_3 x_3 = C^* = \text{const.}$$

Thus, the investigated flows have the following geometrical property: Along each streamline the component u_3 of the velocity vector preserves a constant value. The parametric representations of x_3 along the streamline and the functions g and g_1 have the form

$$x_3 = \frac{C^*}{K_2}\left(\lambda^2 - \frac{\gamma-1}{2}\right)^{-\frac{1}{\gamma-1}} \lambda^{\frac{\gamma+1}{\gamma-1}}; \quad (3.2)$$

$$g = 1 - \frac{\gamma-1}{2}\frac{1}{\lambda^2}, \quad g_1 = \frac{1}{\lambda}. \quad (3.3)$$

Inasmuch as $g > 0$, we deduce the following constraint on the parameter λ from (3.3):

$$\lambda \geqslant \sqrt{2/(\gamma-1)}.$$

The case $K_2 < 0$ is not realized, because it induces the relation $g < 0$. It follows from (3.1) and (3.2) that for $\lambda = \sqrt{(\gamma+1)/2}$ the streamlines have a turning point, while for $\lambda \to \sqrt{2/(\gamma-1)}$ and $\lambda \to \infty$ they go to infinity. We can therefore construct two types of flows: one with $\lambda \in (\sqrt{2/(\gamma-1)}, \sqrt{(\gamma+1)/2}]$ and the other with $\lambda \in [\sqrt{(\gamma+1)/2}, \infty)$. The first, as we infer from (3.3), is a rarefaction flow, and the second is a compression flow. For $\gamma = 3$ the integration in (3.1) is carried to completion. The streamline equations are

$$x_1 = \frac{1}{K_2}\left(\ln\left(\lambda + \sqrt{\lambda^2-1}\right) + \frac{\lambda}{\sqrt{\lambda^2-1}}\right) + C, \quad C = \text{const},$$
$$x_3 = \frac{C^*}{K_2}\frac{\lambda^2}{\sqrt{\lambda^2-1}}. \quad (3.4)$$

For $\gamma = 3$, $K_2 = 1$, and $C = 0$ Fig. 1 shows the streamlines for rarefaction flow I in a semi-infinite plane channel for $\lambda \in [\sqrt{2}, 1)$ (the channel walls correspond to $C^* = 1$ and $C^* = 2$) and for compression flow II for $\lambda \in [\sqrt{2}, \infty)$. In the rarefaction flow the density of the gas decreases to zero at infinity.

We now examine the possibility of the rarefaction flow I going over to compression flow of type II by means of a shock wave under the conditions of the problem stated in Sec. 2. Let the background functions in the rarefaction flow I be described by Eqs. (3.1)-(3.3), $\gamma G_0 = 1$, and for $\lambda^* \in (\sqrt{2/(\gamma-1)}, \sqrt{(\gamma+1)/2})$ let the plane $x_1 = x_1^*$ correspond to a shock wave [under the Hugoniot conditions (2.4) $\Psi_2 = \Psi_t = 0$, $\Psi_1 = 1$]. Expanding the Hugoniot conditions, we have

$$g_1^+ = \frac{2\gamma G_0 g^*}{(\gamma+1) g_1^*} + \frac{\gamma-1}{\gamma+1} g_1^*, \quad (3.5)$$

where the index + corresponds to the parameters of the gas after passage of the shock wave and g^* and g_1^* are determined from (3.3) for $\lambda = \lambda^*$. From (3.5) and (3.3) we obtain for the parameter λ^+

$$\lambda^+ = (\gamma+1)/2\lambda^* > \sqrt{(\gamma+1)/2},$$

i.e., after passage of the shock wave the flow is of type II with $\lambda \in [\lambda^*, \infty)$. Choosing the values of the constants C and K_2 in (3.4) in such a way as to ensure continuity of the streamlines for $x_1 = x_1^*$, we obtain a gas flow with a shock wave $x_1 = x_1^*$ in a semiinfinite plane channel, which first diverges for $x_1 < x_1^*$ and then converges for $x_1 > x_1^*$. For $x_1 > x_1^*$ compression flow is realized, where

$$\rho^+/\rho^* = 2/(\gamma+1)\lambda^{*2} > 1.$$

For $\gamma = 3$ and $\lambda^* = 1.1$ Fig. 1 illustrates such a channel, the numeral III corresponding to the compression zone in the channel.

We now consider isentropic flows B described by Eqs. (1.17) with $G = G_0 = \text{const}$. We construct a special class of solutions for this system, putting

$$g_1 = G_1(\xi),\ g_k = G_k(\xi) + tT_k(\xi),\ l_k = L_k(\xi),\ f_k = F_k(\xi),\ k=2, 3,$$
$$g = G(\xi),\ \xi = x_1 - at,\ a = \text{const},\ H = G_1 - a. \quad (3.6)$$

Substituting (3.6) into (1.17) and setting the coefficients of t and the free terms equal to zero, we obtain a system of ten ordinary differential equations with independent variable ξ in ten functions G, H, G_k, T_k, L_k, F_k of the form

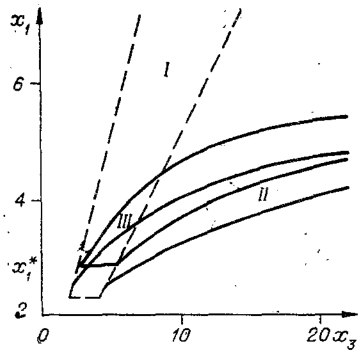

Fig. 1

$$
\begin{gathered}
HH' + \frac{1}{\gamma-1} G' = 0, \quad HL_k' + L_2 L_k + L_3 F_k = 0, \\
HF_k' + F_2 L_k + F_3 F_k = 0, \quad HG_k' + G_2 L_k + G_3 F_k + T_k = 0, \\
HT_k' + T_2 L_k + T_3 F_k = 0, \quad \frac{1}{\gamma-1} HG' + GH' + G(L_2 + F_3) = 0.
\end{gathered} \tag{3.7}
$$

Specifying the initial data for the system (3.7), such that

$$
H(\xi_0) \neq 0, \ H^2(\xi_0) - G(\xi_0) \neq 0, \tag{3.8}
$$

and solving the system (3.7) for the derivatives, we find that the Cauchy problem formulated for $\xi = \xi_0$, given conditions (3.8), has a unique solution in the neighborhood of the point ξ_0.

The components of the velocity vactor and the sound velocity squared θ now take the form

$$
\begin{gathered}
u_1 = G_1(\xi), \ \theta = \theta(\xi), \\
u_k = G_k(\xi) + tT_k(\xi) + x_2 L_k(\xi) + x_3 F_k(\xi), \ k = 2,3.
\end{gathered} \tag{3.9}
$$

The system (3.7) generates a class of unsteady rotational gas flows. These solutions are important in the following aspect. The class of flows with a degenerate velocity hodograph, where the four-dimensional domain of definition of the flow in the physical x_1, x_2, x_3 space corresponds in u_1, u_2, u_3, θ space to a manifold of fewer dimensions, plays a definite role in gasdynamics. It is clear that the solutions (3.9) have a degenerate hodograph, because u_1 and θ are functionally dependent. Calculating the determinant

$$
I = \frac{D(u_1, u_2, u_3)}{D(x_1, x_2, x_3)} = H'(L_2 F_3 - L_3 F_2),
$$

we verify that if $H'(\xi_0) \neq 0$, $(L_2F_3 - L_3F_2)(\xi_0) \neq 0$ for $\xi = \xi_0$, then the corresponding flow is a triple shock [5].

In the potential case [5] the construction of nontrivial solutions in the class of triple shocks requires the analysis of a cumbersome overdetermined system of partial differential equations. Only one substantial class of such solution is known [6].

It follows from (3.9) that the level manifold of all gasdynamical variables comprises straight lines in x_1, x_2, x_3, t space. We thus infer from (3.9) that such a manifold is determined by the intersection of three hyperplanes (one of which is described by the equation ξ = const). In this case the straight level lines under the conditions $G_1 \not\equiv 0$, $I \not\equiv 0$, generally speaking, do not pass through a common fixed point of x_1, x_2, x_3, t space, i.e., the flow is not conical. Consequently, the constructed solution is a nonconical rotational triple shock with rectilinear generatrices.

It is essential to test the solution for nonconicity, because the existence of self-similar conical rotational triple shocks is a trivial direct consequence of the gasdynamical equations. The type of solutions constructed here proves the nonemptiness of the class of nonconical rotational triple shocks (a complete description of this class of flows does not exist at the present time).

Flows having rectilinear level lines have been classified for the unsteady planar case in [7] and for the steady three-dimensional case in [8]. There is no such classification for unsteady three-dimensional flows. In this connection, it has been found in the cases investigated above that nonconical rotational flows of the given classes — double shocks — exist only in the exceptional case of an adiabatic exponent $\gamma = 2$. The constructed solution (3.9) shows that this situation does not occur in unsteady three-dimensional flow, while nonconical rotational triple shocks with rectilinear level lines exist for any values of γ.

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MODELS AND SIMILARITY ANALYSIS IN THE THERMODYNAMICS OF GAS—LIQUID SYSTEMS

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Fundamental similarity numbers are considered for heat transfer and dynamics in gas—liquid systems, including effects due to change of state and loss of stability.

Introduction. Mikhail Alekseevich Lavrent'ev is a great master who has produced clear and efficient physicomathematical models in hydrodynamics.

There are numerous relevant factors in the thermohydrodynamics of multiphase systems, and the flows have a multiplicity of structure, so such models are particularly important. Therefore, we hope that a compact exposition of some results in this area will constitute a tribute.

Some of the complicated problems in thermokinetics and mechanics of flowing media arise in the dynamics of gas—liquid systems, particularly in energy transport in phase transitions.

The following features are the most important:

1. The variety of dynamic structures and the variability in the spontaneous formations (bubbles, droplets, films, and jets) in space and time.

2. The wave effects at interfaces and within mixtures related to surface tension and the substantial dependence of signal transmission speeds on component concentrations and element structures.

3. Effects from the thermohydrodynamics of primary nucleation and the distribution of condensation centers at boundaries and within the flow.

4. The scope for states essentially metastable in the thermodynamic sense.

5. The complications of turbulent transport related to features of the flow in the elements of each phase, in addition to the common interphase turbulence.

6. Quasiturbulent states can occur in a laminar flow on account of oscillations of dispersed elements of the other phase.

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